

# Top-of-Climb Matching Method for Reducing Aircraft Trajectory Prediction Errors

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The inaccuracies of the aircraft performance models utilized by trajectory predictors with regard to takeoff weight, thrust, climb profile, and other parameters result in altitude errors during the climb phase that often exceed the vertical separation standard of 1000 feet. This study investigates the potential reduction in altitude trajectory prediction errors that could be achieved for climbing flights if just one additional parameter is made available: top-of-climb (TOC) time. The TOC-matching method developed and evaluated in this paper is straightforward: a set of candidate trajectory predictions is generated using different aircraft weight parameters, and the one that most closely matches TOC in terms of time is selected. This algorithm was tested using more than 1000 climbing flights in Fort Worth Center. Compared to the baseline trajectory predictions of a real-time research prototype (Center/TRACON Automation System), the TOC-matching method reduced the altitude root mean square error (RMSE) for a 5-minute prediction time by 38%. It also decreased the percentage of flights with absolute altitude error greater than the vertical separation standard of 1000 ft for the same look-ahead time from 55% to 30%.

## I. Introduction

AIR traffic demand is expected to increase about 50 percent during the next 20 years [1], but air traffic controller workload is one of the primary factors that is predicted to constrain airspace capacity. As such, it is expected that higher levels of automation for separation assurance are needed to accommodate future demand growth. Previous research showed that climb trajectory prediction errors [2-9] must be reduced from current levels [5] because they cause late conflict detections [6-9] that limit the safety and efficiency that can be provided by such automation. The top-of-climb (TOC) matching method described and evaluated in this paper focuses on improving vertical trajectory prediction accuracy for climbing flights because 50-75% have altitude errors that exceed the 1000-foot (ft) vertical

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separation standard for trajectory prediction look-ahead times greater than 3 minutes [9]. In current operations, air traffic controllers compensate for climb uncertainty by issuing temporary altitude maneuvers to direct flights to stop climbing at an altitude below their desired, optimal cruising altitudes. These maneuvers increase both fuel burn and communication frequency congestion and could be avoided if the trajectory predictions for climbing flights were more accurate (especially in the vertical dimension) [10]. The TOC-matching method does not focus on improving horizontal trajectory prediction accuracy for climbing flights because only about one percent have horizontal errors that exceed the 5-nautical mile (nmi) horizontal separation standard [9, 11].

Researchers have improved climb trajectory prediction accuracy by adjusting aircraft performance models based on publicly available flight manuals [12] and analyzing trajectory errors [13]. The authors of the first study utilized model parameter identification to determine the combinations of thrust and drag that best modeled the time-to-TOC data specified in the Boeing 737-400 flight manual. The average difference in time to TOC between the trajectories generated by the authors using their enhanced Boeing 737-400 model and the Boeing aircraft performance software was three seconds or less. However, the authors did not evaluate their improved model with actual flights. The authors of the second study computed the along-track, cross-track, and altitude trajectory prediction errors for actual climbing Boeing 737, Boeing 757, and McDonnell-Douglas MD-80 flights (comprising about 84% of large aircraft) in Fort Worth Center. They adjusted the weight, thrust, and speed parameters in their performance models for these aircraft types based on a few representative examples of each until the errors approached zero. When they applied their adjusted aircraft performance models to the full set of climbing flights, they observed a reduction in the altitude errors for large aircraft types during the climb phase with regard to both mean (from 713 ft to 36 ft) and standard deviation (from 1709 ft to 1180 ft). However, they did not apply their method to other prominent aircraft classes, such as regional jets, which comprise about 30% of departures in Fort Worth Center and had mean altitude errors of as much as 3200 ft [7].

Previous research also demonstrated that climb trajectory prediction accuracy could also be improved through the use of airline flight-planning data [14] and real-time air-to-ground data link of flight parameters [15]. In the first study, the author gave two examples where estimated aircraft gross takeoff weight data from Airline Operations Centers resulted in more accurate climb trajectory predictions. However, the author also stated that the predictions for some flights actually became less accurate possibly due to errors in aircraft thrust performance models. Since the aggregate results were not reported, it is not clear how much improvement can be expected for climbing flights in

general. In the second study, the use of flight parameters, such as aircraft weight and climb speed schedule, acquired via air-to-ground data link reduced the mean altitude error for climbing flights in half. On the other hand, this result was based on just twenty Boeing 777 flights that were specifically selected for the analysis because large errors were observed in their flight parameters. Thus, it is not certain that the same level of improvement would be realized for the full range of aircraft types and climb profiles that are present in actual operations.

Recent work on adaptive thrust [16-17] and adaptive weight [9, 11] algorithms also demonstrated substantial improvements in trajectory prediction accuracy for actual climbing flights. Both algorithms only used the radar track and atmospheric forecast data available in current operations to dynamically adjust the modeled aircraft thrust or weight, respectively, on a per-flight basis. The adapted trajectory predictions more closely matched actual flight paths. Overall, the adaptive weight algorithm reduced both altitude and TOC time root mean square errors (RMSE) by about 20% for a data set comprised of about 400 actual Fort Worth Center climbing departures [11]. No aircraft types or climb profiles were intentionally included or excluded in that analysis. Although complementary fast-time simulations showed that having additional climb profile data can decrease the residual trajectory prediction errors by more than half and the missed and false alert rates by roughly 15 percentage points [9], this information is currently unavailable and will continue to be unavailable unless aircraft are mandated to share such proprietary flight data in the Next Generation Air Transportation System (NextGen).

This paper demonstrates the potential improvements in trajectory prediction accuracy for climbing flights in the vertical dimension that could be realized if just one additional parameter is made available: TOC time.<sup>†</sup> Compared to flight-specific aircraft weight, thrust, and climb profile data that are proprietary and were shared by airlines in prior research [14-15] only under special arrangements, airlines are more likely to share information on TOC, which can be identified in radar track data that are publicly available. It should be noted that TOC time is used instead of TOC horizontal position because air traffic controllers need to have reliable predictions of the time it will take for flights to climb to their flight plan altitudes when performing conflict detection and resolution to maintain safe separation between flights [10], which is less likely if the latter is used. TOC time may also be used in conjunction with other flight parameters depending on the tradeoff between trajectory prediction accuracy at TOC and the rest of the climb phase, but this line of investigation is beyond the scope of this paper.

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<sup>†</sup> An earlier version of this work was presented at the 2013 AIAA GNC Conference [18]. It was revised and extended substantially based on feedback received during and after the conference.

Compared to methods that utilize the same adjusted aircraft performance model for all flights of the same aircraft type [12-13, 19], the TOC-matching method proposed in this study is more effective because it is applied on a per-flight basis like the adaptive weight algorithm [9, 11]. While the adaptive weight algorithm studies demonstrated the improvement in climb trajectory prediction accuracy that can be achieved using currently available data, this paper on the TOC-matching method demonstrates what may be possible in the near future. TOC data is already calculated by the on-board Flight Management System (FMS) and could potentially be shared in NextGen through Automatic Dependent Surveillance-Broadcast (ADS-B) Out [20-21]. One reason why flights do not share TOC data in current operations is because the FMS and the corresponding databus would need to be enhanced and certified to export this data. Speculating on how policymakers and stakeholders could weigh the costs of these enhancements against the benefits of improved climb trajectory prediction accuracy on both safety and efficiency is beyond the scope of this study, though.

The remainder of this paper is organized as follows. The next section describes the TOC-matching method and illustrates how it substantially improved altitude trajectory prediction accuracy for a typical climbing departure in Fort Worth Center. An analysis of the trajectory predictions with and without the TOC-matching method for over 1000 actual Fort Worth Center climbing departures from 14 days in February 2008 is then presented and discussed at both an aggregate level and by aircraft type. The results of a closer investigation of Embraer E145 departures are then shown because the TOC-matching method improved the prediction accuracy of these flights the least out of the ten most common aircraft types in Fort Worth Center. Following that is a discussion on why the TOC-matching method can be utilized with any trajectory predictor, including the FAA's En Route Automation Modernization Trajectory Predictor (or ERAM TP) [22], and why it is expected to improve climb trajectory prediction accuracy in any airspace and for all aircraft types. Then, the sensitivity of the algorithm is analyzed with respect to: 1) TOC time accuracy since the FMS-predicted TOC times that flights broadcast are unlikely to be their actual TOC times, and 2) fewer candidate trajectory predictions (e.g., due to computational constraints). Lastly, the findings of the research are summarized.

## **II. TOC-Matching Method**

The trajectory predictor evaluated in this study is the Center/TRACON Automation System (CTAS) Trajectory Synthesizer (TS) [23] that was analyzed in prior work [6-9, 11-15]. CTAS is a real-time research prototype system

developed at NASA that includes mature capabilities for 4-D trajectory prediction, conflict detection, conflict resolution, and other functions [24]. The CTAS TS generates trajectory predictions based on enroute Center Host flight plan and radar track data and atmospheric condition forecasts (e.g., wind, temperature) from the National Oceanic and Atmospheric Administration Rapid Update Cycle model (which was succeeded by Rapid Refresh starting in May 2012). Baseline CTAS trajectories are calculated using a simplified form of the point-mass equations of motion [23] with baseline weight equal to 90% of the maximum gross takeoff weight in the aircraft type-specific CTAS performance model database.

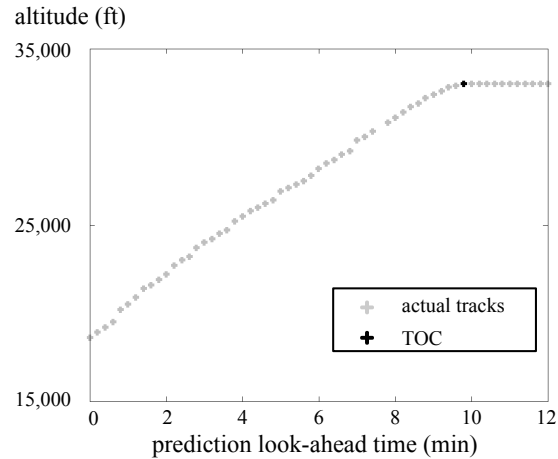
Table 1 contains terminology that will be used throughout the rest of this paper to describe the TOC-matching method and the analysis of CTAS TS trajectory predictions with and without the use of this algorithm.

**Table 1 Terminology for trajectory prediction analysis**

Phrase/word	Definition
Baseline trajectory	Baseline CTAS trajectory prediction
Baseline weight	Modeled weight parameter used to generate the baseline trajectory prediction
TOC	Top of climb
TOC-matching trajectory	Trajectory prediction with TOC time closest to the TOC time provided
TOC-matching weight	Modeled weight parameter corresponding to the TOC-matching trajectory
Weight percentage	Percentage of the maximum gross takeoff weight in the CTAS performance model

## A. Description

The TOC-matching method proposed in this paper searches for a trajectory prediction that is closest to the TOC time provided. In actual operations, TOC time calculated by on-board FMS could potentially be broadcast via ADS-B Out. However, since these data are not currently available, this study uses the first time at which the altitude of the flight in the recorded track data equaled its filed flight plan altitude as a proxy. This is illustrated in Figure 1 for an actual climbing flight in Fort Worth Center. A sensitivity analysis of the TOC-matching method to TOC time data accuracy is also performed in this study (see Section V.B) because the FMS-predicted TOC times are unlikely to be their actual TOC times.



**Fig. 1 Illustration of actual TOC identified for an actual Fort Worth Center departure.**

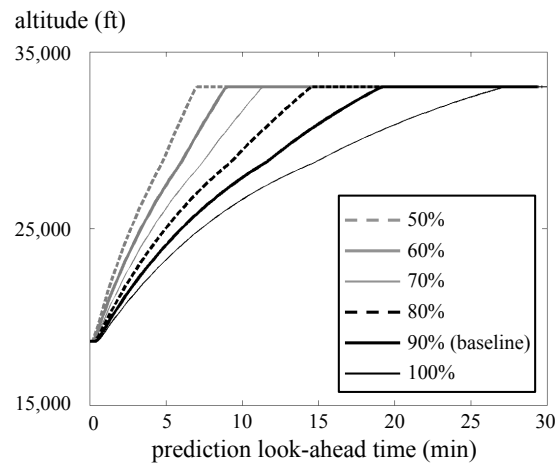
The first step in the TOC-matching method is to compute candidate trajectory predictions for climbing flights. In this study, 51 candidate trajectory predictions were generated for each climbing flight at each radar track during the climb phase of flight using weight percentages that ranged between 50% and 100% of the maximum gross takeoff weight in the aircraft type-specific CTAS performance model database in increments of one percentage point. Each set of predictions spanned a wide range of climb profiles in the vertical dimension that were expected to contain the actual climb profile of the flight.

The second step in the TOC-matching method is to identify TOC in each of the candidate trajectory predictions by finding the first time at which the predicted altitude equaled the filed flight plan altitude. The absolute difference relative to the actual TOC time (which, in actual operations, would be the TOC time computed by the flight's FMS) is then calculated for each of the 51 candidate trajectory predictions. The one with the smallest absolute difference (i.e., the one that most closely matches the flight's TOC time) is the TOC-matching trajectory prediction, and the corresponding modeled weight is the TOC-matching weight. Note that a large absolute difference between the actual and predicted TOC times will result in large climb trajectory prediction errors that persist even as flights get closer to TOC.

## **B. Application to a Representative Fort Worth Center Departure**

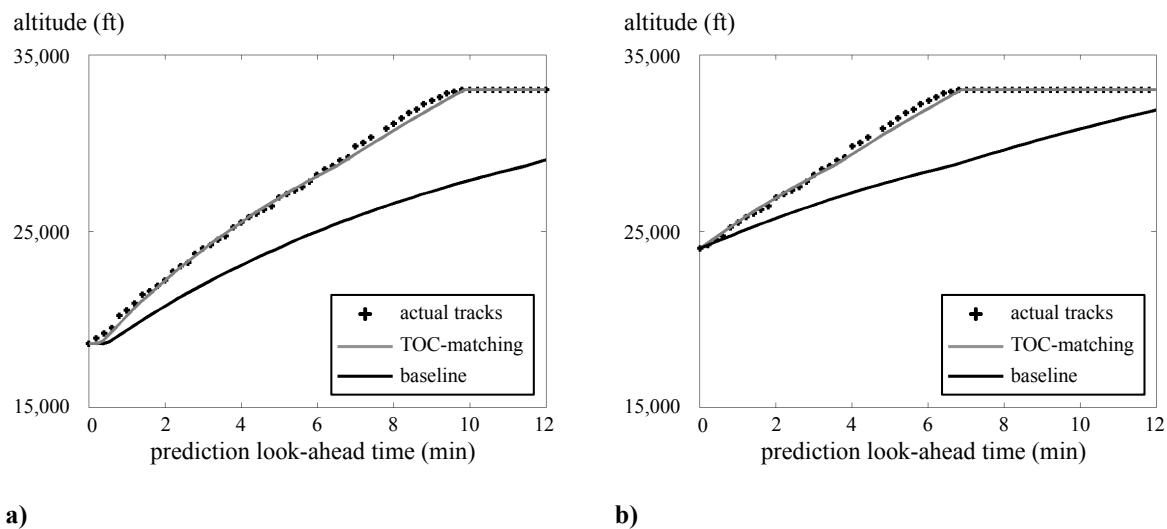
The remainder of this section will demonstrate the TOC-matching method for the same actual Fort Worth Center departure plotted in Figure 1. A subset of the 51 candidate trajectory predictions that were generated for the first step

of the algorithm when this flight was at 18,000 ft is illustrated in Figure 2. The leftmost prediction with the earliest TOC time was computed using the lowest weight percentage (50%) while the rightmost one with the latest TOC time was computed using the highest weight percentage (100%). The TOC of these candidate trajectory predictions and each of the 45 other candidate trajectory predictions were then identified. For this flight, the candidate trajectory prediction with predicted TOC that was closest to the actual TOC (i.e., the TOC-matching trajectory) was computed using an aircraft weight percentage of 64% (i.e., the TOC-matching weight).



**Fig. 2 Subset of the 51 candidate trajectory predictions computed for an actual Fort Worth Center departure at 18,000 ft.**

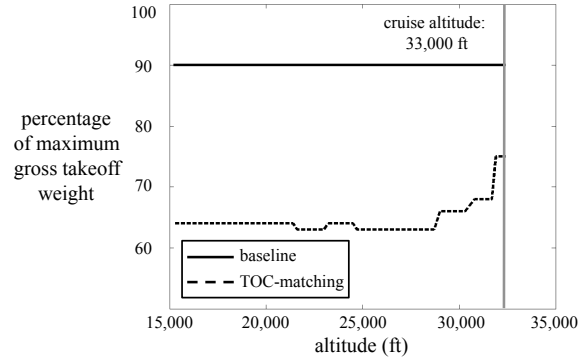
Figures 3a and 3b plot the actual altitude track data (+'s), baseline CTAS trajectory predictions (solid black lines), and TOC-matching trajectory predictions (solid gray lines) for the same flight at 18,000 ft and 24,000 ft, respectively. Note how the baseline trajectory predictions have a slower vertical rate than what the aircraft actually flew. At 18,000 ft, the altitude errors of the baseline trajectory prediction exceed the vertical separation standard of 1000 ft within 1 minute. By comparison, the TOC-matching trajectory prediction closely matches the actual track data. Figure 3b demonstrates that the TOC-matching method maintains this improvement later in the climb phase.



**Fig. 3** Example TOC-matching trajectory predictions at a) 18,000 ft, and b) 24,000 ft that closely match the track data of an actual Fort Worth Center departure.

At 18,000 ft (see Figure 3a), the TOC-matching weight percentage was 64%, which substantially differs from the baseline weight percentage of 90%. The TOC-matching weight percentages for this flight do not vary much during most of the climb phase (see Figure 4). This is essential because inconsistent trajectory predictions could result in erratic conflict prediction information that air traffic controllers would consider to be unreliable. The TOC-matching weight percentage is either 63% or 64% until the flight climbs above 29,000 ft, at which point it increases modestly as the flight approaches its cruise altitude of 33,000 ft except for a sharp increase to 75% just before TOC. However, this is acceptable because the TOC-matching trajectory predictions at consecutive tracks during the latter portion of this flight's climb are not substantially different because it is already close to TOC.





**Fig. 4 TOC-matching weights during the climb phase of an actual flight in Fort Worth Center.**

It is important to point out that the altitude errors of the TOC-matching trajectories may continue to be larger than desired even as the flight approaches TOC. This is because the TOC-matching method was constrained to only generate candidate trajectory predictions using weight percentages that ranged between 50% and 100% of the maximum gross takeoff weight in the aircraft type-specific CTAS performance model database in increments of one percentage point. For this and other reasons (explored in Sections IV and V), the TOC-matching method is not able to fully account for all of the sources of climb trajectory prediction uncertainty that are present in actual operations. Additional work to investigate the effect of expanding the set of parameters (e.g., climb speed schedule) and their respective ranges that can be used by the TOC-matching method is important, but beyond the scope of this study.

It should be emphasized that the TOC-matching method does not estimate actual aircraft weight [25] or fuel burn [26] even though TOC-matching weights are identified. In fact, due to the wide range of sources of uncertainty that cause climb prediction errors, the algorithm may even select a candidate trajectory whose modeled aircraft weight differs substantially from the true aircraft weight. Rather, the algorithm simply seeks out the candidate trajectory prediction whose TOC most closely matches the given TOC data with the implicit expectation that the trajectory predictor will then model the rest of the climb trajectory more accurately. This paper analyzes the baseline and TOC-matching trajectory predictions for over 1000 actual Fort Worth Center departures to determine the extent to which this expectation is true.

The corresponding TOC-matching weight is just a by-product of the algorithm. The TOC-matching method will never be able to completely compensate for all sources of climb uncertainty, which means that the resulting TOC-

matching trajectory predictions will never perfectly match the actual radar track data. It simply improves overall climb trajectory prediction accuracy (see Sections IV and V).

### III. Trajectory Prediction Analysis Methodology

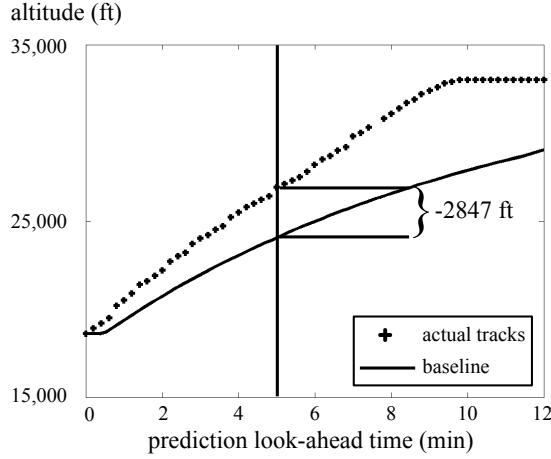
Baseline and TOC-matching method trajectory predictions were generated using the CTAS TS for all Fort Worth Center departures during 14 days in February 2008. However, trajectory prediction errors were computed for a flight only if it met two conditions from the time that the trajectory prediction was made until the time that the flight plan cruise altitude was achieved in either the track or the trajectory prediction data (whichever came first): 1) no flight plan amendments, and 2) no non-climb segments. The first condition is intended to remove the effect of controller intervention from the analysis. Only about 10% of Fort Worth Center departures (roughly 1500 flights) met the first condition. The second condition limits the analysis to the climb portion of flight that is the focus of this study.

#### A. Vertical Dimension

An example of the trajectory prediction analysis in the vertical dimension is illustrated in Figure 5 for the same flight presented in Section II. Altitude ( $h$ ) trajectory prediction errors were computed as a function of look-ahead time  $t$  with the track data serving as the reference:

$$h_{error}(t) = h_{pred}(t) - h_{track}(t) \quad (1)$$

Figure 5 illustrates this calculation for the trajectory prediction generated at the first track above 18,000 ft. This allowed the flight about 4 minutes to achieve a steady climb speed following the airspeed restriction of 250 knots at 10,000 ft. The results section will also include analysis of the trajectory predictions computed at the first track above 24,000 ft to determine the extent to which the TOC-matching method can still improve climb trajectory prediction accuracy when flights are closer to TOC.



**Fig. 5 Example altitude trajectory prediction error calculation.**

In this case, the altitude trajectory prediction error for a look-ahead time of 5 minutes was -2847 ft because the predicted altitude was 24,053 ft while the actual track altitude was 26,900 ft. The altitude errors for this flight were calculated only up to about a 10-minute prediction look-ahead time when the actual track reached the flight plan cruise altitude of 33,000 ft. On the other hand, if the trajectory prediction reached 33,000 ft before the track data, then the analysis would have been performed only up to that earlier prediction look-ahead time. This methodology will lead to differences in the number of baseline and TOC-matching trajectory predictions that are evaluated at any given prediction time in general (and especially at longer look-ahead times) because it depends on the accuracy of the trajectory predictions.

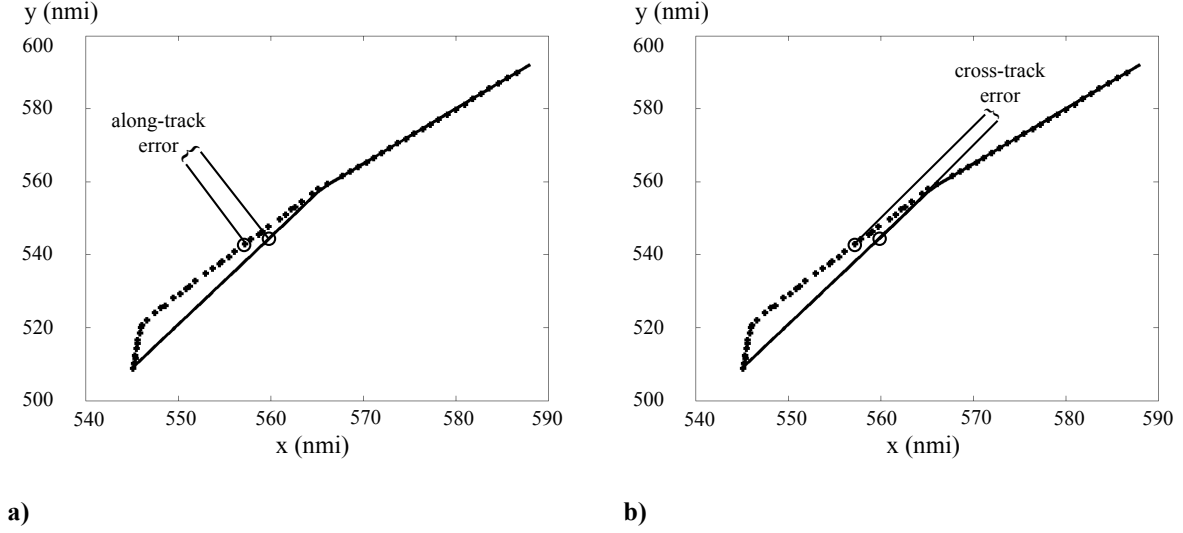
## B. Horizontal Dimension

Two measures of horizontal trajectory prediction error were also computed for the completeness of the analysis even though the TOC-matching method is not expected to substantially affect the horizontal dimension because the candidate trajectory predictions were generated using different modeled aircraft weights (instead of different climb speeds, for example) [7]. Along-track errors  $[a_{error}(t)]$  and cross-track errors  $[c_{error}(t)]$  were calculated parallel and perpendicular, respectively, to the predicted course with the track data serving as the position reference:

$$a_{error}(t) = [x_{pred}(t) - x_{track}(t)] \cdot \sin(\psi_{pred}(t)) + [y_{pred}(t) - y_{track}(t)] \cdot \cos(\psi_{pred}(t)) \quad (2)$$

$$c_{error}(t) = [x_{pred}(t) - x_{track}(t)] \cdot \cos(\psi_{pred}(t)) - [y_{pred}(t) - y_{track}(t)] \cdot \sin(\psi_{pred}(t)) \quad (3)$$

Figures 6a and 6b illustrate how these errors were calculated for the same flight at the same 5-minute prediction look-ahead time as in Figure 5 for the altitude trajectory prediction error calculation. In this case, the along-track error was 0.89 nmi, and the cross-track error was 0.94 nmi.



**Fig. 6 Example a) along-track error, and b) cross-track error calculations.**

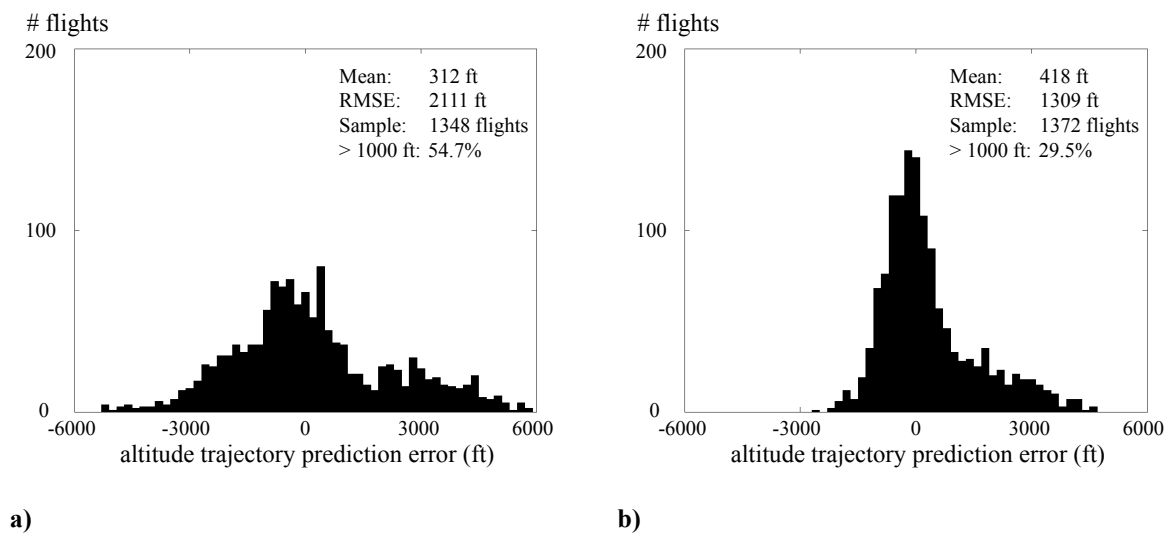
#### IV. Results

This section contains analysis of the baseline CTAS TS and TOC-matching trajectory predictions for more than 1000 Fort Worth Center departures over 14 days in February 2008. The same traffic set was analyzed in an earlier study [7]. The primary summary metric used to compare the results is root mean square error (RMSE). It is a good measure because it places relatively higher weight on larger errors, which are particularly undesirable for trajectory predictions since they are used in separation assurance to keep aircraft safely separated. However, some flights will have altitude errors that exceed the vertical separation standard of 1000 ft even when RMSE is less than 1000 ft. As such, the percentage of climbing flights whose absolute altitude trajectory prediction error is greater than 1000 ft is a complementary metric that will also be reported. Note that all reductions in trajectory prediction error achieved by the TOC-matching method in this study are in addition to what was already attained in prior research [12-13] that improved the aircraft performance models used by the CTAS TS.

## A. Aggregate Results

### 1. Altitude Errors

Figures 7a and 7b are histograms of altitude errors for the baseline and TOC-matching trajectory predictions, respectively, that were generated at the first track update above 18,000 ft. Recall that this altitude threshold allowed flights about 4 minutes to achieve a steady climb speed following the 250-kt speed restriction at 10,000 ft (assuming a typical vertical rate of 2000 ft/min).



**Fig. 7 Altitude errors for a) baseline, and b) TOC-matching trajectory predictions generated at 18,000 ft (aggregate, 5-minute prediction time).**

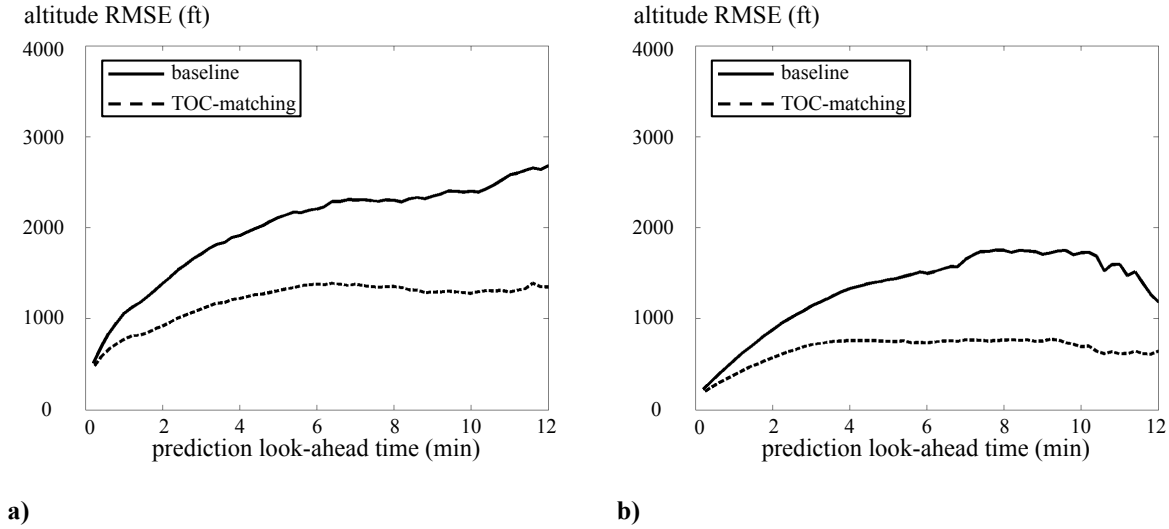
The baseline trajectory predictions had altitude RMSE of 2111 ft. Similar analysis of the trajectory predictors used in the Federal Aviation Administration's User Request Evaluation Tool (URET) and En Route Automation Modernization (ERAM) found comparable levels of altitude trajectory prediction errors [5]. By comparison, the TOC-matching trajectories had altitude RMSE of 1309 ft. Although this exceeds the vertical separation standard of 1000 ft, it is still 38% smaller than the altitude RMSE of the baseline trajectory predictions. Note that this is nearly double the 20% reduction in altitude RMSE that was achieved by the adaptive weight algorithm when it was also applied to the CTAS TS trajectory predictor for actual Fort Worth Center departures [11] (albeit for different days).

The TOC-matching method was particularly successful at reducing the number of flights at the tails of the error distribution, especially on the negative (left) side. The persistence of the errors on the positive (right) side indicates

that the slowest candidate climb trajectory prediction (generated using 100% of the maximum gross takeoff weight) had a faster climb profile than the actual flown track in many cases. In any case, the percentage of flights that had absolute altitude error greater than the vertical separation standard of 1000 ft dropped from 54.7% in the baseline to 29.5% when the TOC-matching method was used. Although the level of climb trajectory prediction accuracy that is necessary for higher levels of automation in separation assurance is an open research question, these results indicate that the TOC-matching method can achieve substantial improvement towards that standard.

Note that 24 (or about 1.8%) more flights were included in the TOC-matching analysis compared to the baseline analysis even though the trajectories for both were generated concurrently for the Fort Worth Center traffic data set. Recall that this is because trajectory prediction errors were calculated only up to the point where either the track data or the trajectory prediction achieved the flight plan cruise altitude (see Section III.A). Thus, the altitude trajectory prediction errors for flights with predicted TOC times that were less than 5 minutes in the future were not included in Figures 7a and 7b. Since the TOC-matching trajectory predictions tended to be more accurate than the baseline trajectory predictions, more flights met the criteria to be included in the TOC-matching error histogram.

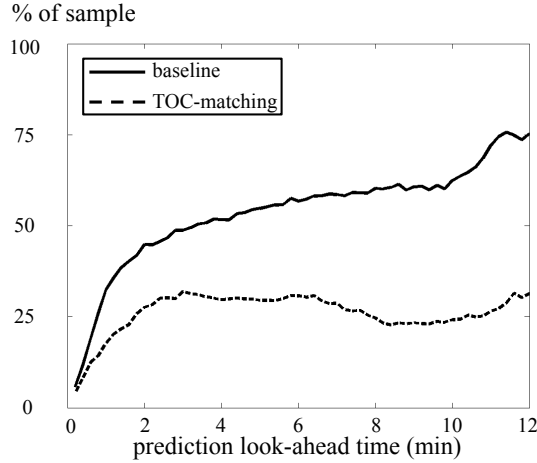
Similar improvements in trajectory prediction accuracy for climbing flights were also observed more generally: 1) across all prediction look-ahead times, and 2) at different altitudes throughout the climb phase. For instance, in Figure 8a, the amount of improvement in altitude RMSE generally increases as a function of look-ahead time for the trajectory predictions generated at 18,000 ft. Figure 8b is a similar chart for trajectory predictions made at 24,000 ft that illustrates how the TOC-matching method generally maintained this level of improvement throughout the climb phase even as flights approached TOC. However, it is evident that the TOC-matching method could be enhanced to further reduce the trajectory prediction errors toward zero. These results indicate that the TOC-matching method may need to be modified to use a wider range of weight percentages and/or additional flight parameters (e.g., climb speed schedule). This will be investigated to some extent in the analysis by aircraft type and the discussion section. These results also point to a potential need for additional flight data besides TOC to be made available.



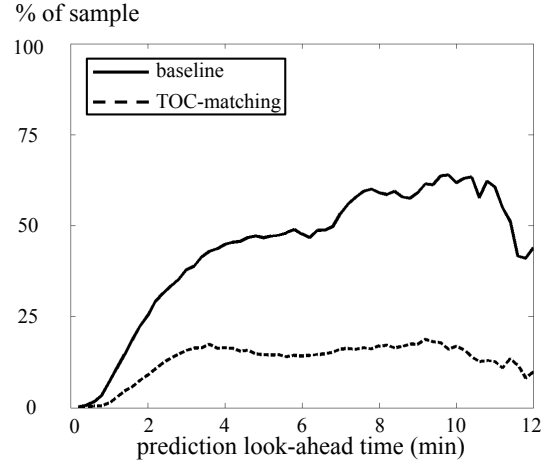
**Fig. 8 Altitude RMSE as a function of look-ahead time for trajectory predictions generated at a) 18,000 ft, and b) 24,000 ft.**

Note that altitude RMSE levels off and declines modestly in the 24,000-ft plot (Figure 8b) at longer look-ahead times, especially in the baseline case. This is contrary to the generalization that uncertainty causes prediction errors to grow over time. The behavior in Figure 8b is due to the trajectory prediction analysis method used (see Section III.A) that calculates errors only up to the point where either the track or the trajectory prediction achieved the flight plan cruise altitude. A trajectory prediction with predicted TOC time that is earlier than what is observed in the track data will be analyzed at shorter look-ahead times but not at longer look-ahead times beyond the predicted TOC time. The trajectory predictions with the largest errors are less likely to be included in the analysis at longer look-ahead times when flights are close to TOC. This is why the altitude RMSE for the baseline trajectory predictions levels off and decreases as a function of prediction look-ahead time in the 24,000-ft case but not in the 18,000-ft case.

The percentage of flights with absolute altitude error greater than the vertical separation standard of 1000 ft also decreased substantially as expected when the TOC-matching method was used. This is illustrated in Figures 9a and 9b where the reduction was as much as 47 percentage points in the 18,000-ft case and 49 percentage points in the 24,000-ft case.



a)



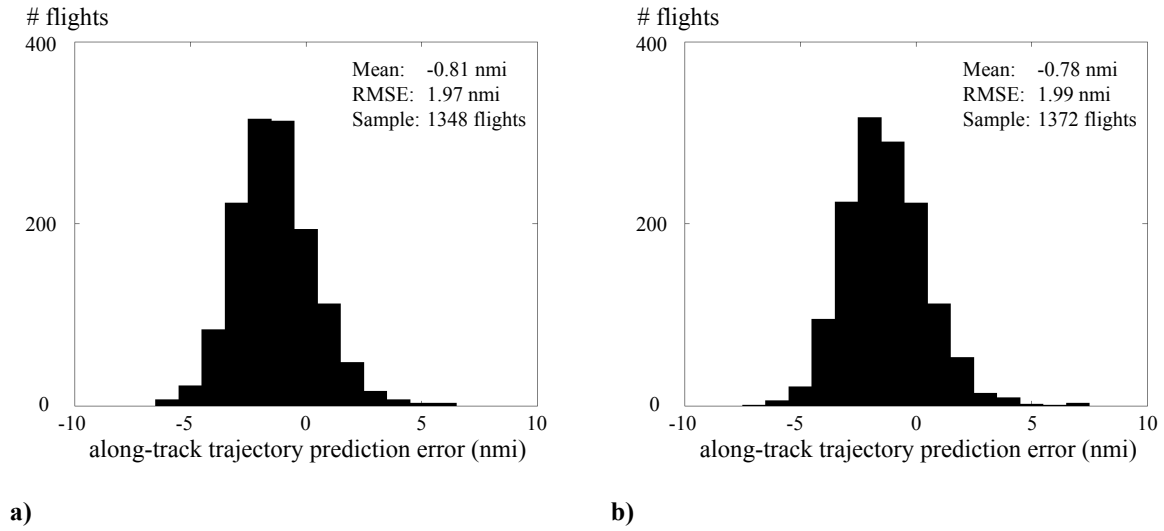
b)

**Fig. 9 Percentage of analysis sample with altitude error greater than 1000 ft as a function of look-ahead time for trajectory predictions generated at a) 18,000 ft, and b) 24,000 ft.**

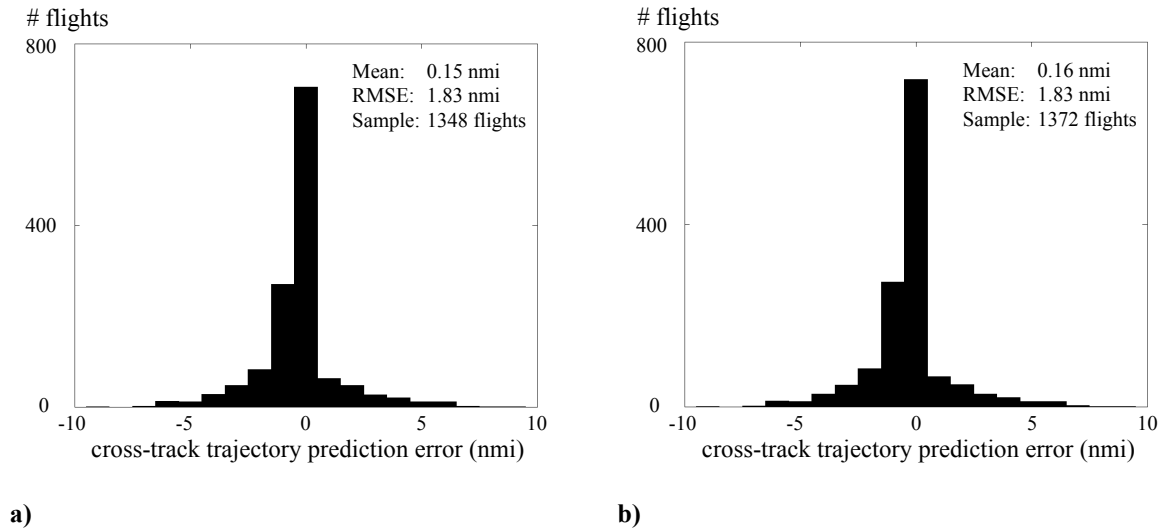
## 2. Horizontal Errors

While the TOC-matching method substantially improved trajectory prediction accuracy for climbing flights in the vertical dimension, it had little effect in the horizontal dimension. Figures 10a and 10b are histograms of along-track errors for the same baseline and TOC-matching trajectory predictions, respectively, that were generated at the first track update above 18,000 ft and analyzed in the previous section. The mean and RMSE were very similar at about -0.80 nmi and 2.00 nmi, respectively, for both sets of trajectory predictions. Similar results were observed for the cross-track errors in Figures 11a and 11b. The RMSE for the baseline and TOC-matching trajectory predictions was 1.83 nmi in both cases with mean error of 0.15 nmi for the former and 0.16 nmi for the latter.





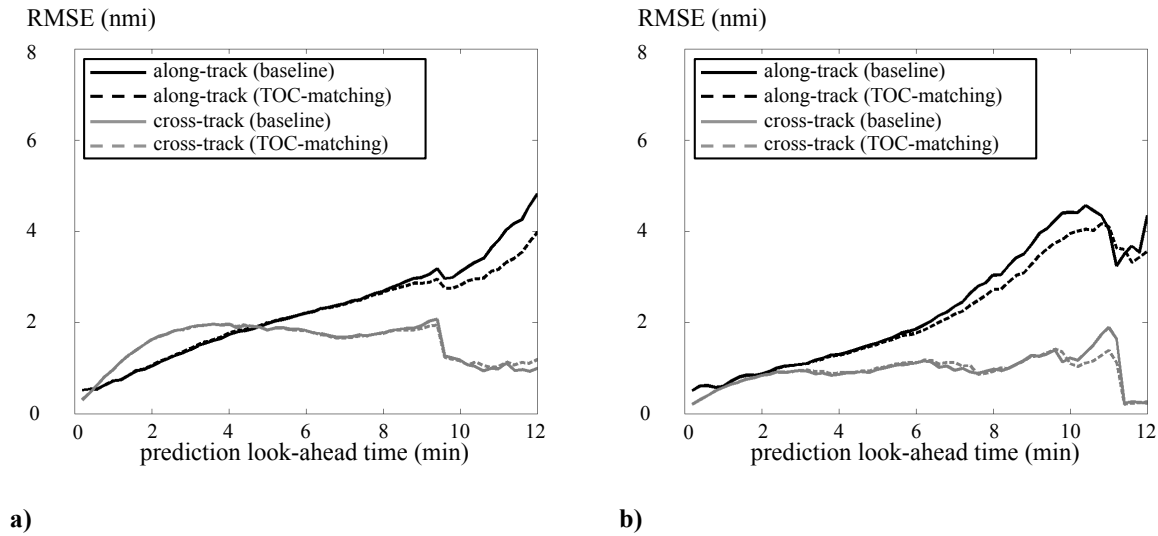
**Fig. 10** Along-track errors for a) baseline, and b) TOC-matching trajectory predictions generated at 18,000 ft (aggregate, 5-minute prediction time).



**Fig. 11** Cross-track errors for a) baseline, and b) TOC-matching trajectory predictions generated at 18,000 ft (aggregate, 5-minute prediction time).

As seen in Figures 12a and 12b, the similarities between the results of the baseline and TOC-matching along-track and cross-track errors were also observed more generally across trajectory prediction look-ahead times and different altitudes throughout the climb phase of flight. These results indicate that other approaches are needed to

reduce horizontal trajectory prediction error to the level required for robust separation assurance: 1) expanding the TOC-matching method to use flight parameters besides weight (e.g., climb speed schedule) to generate candidate trajectory predictions, 2) adjusting the aircraft performance models used by the trajectory predictor, and/or 3) using alternative aircraft performance models. Given the similarity of the along-track and cross-track errors for the two sets of trajectory predictions, the remainder of this paper will focus on the differences in altitude error.

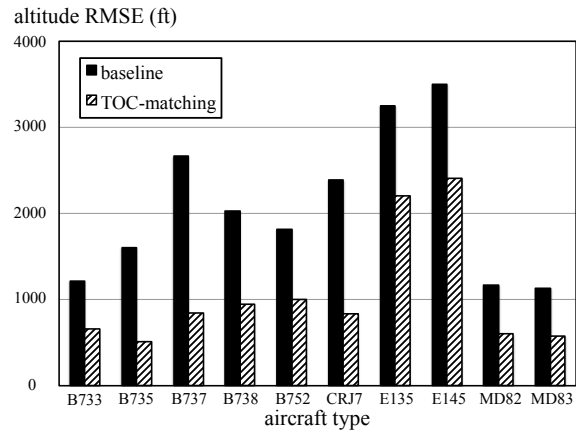


**Fig. 12** Along-track and cross-track RMSE as a function of look-ahead time for trajectory predictions generated at a) 18,000 ft, and b) 24,000 ft.

## B. Altitude Error Results by Aircraft Type

### 1. RMSE

The aggregate-level results of the previous section are encouraging because they demonstrate that TOC data can be used to substantially improve overall altitude trajectory prediction accuracy for climbing flights. A closer look by aircraft type is also needed to determine whether or not this improvement was evenly distributed. Figure 13 is a plot of 5-minute altitude RMSE by aircraft type for the same baseline and TOC-matching trajectory predictions made at the first track above 18,000 ft that were analyzed in the previous section. The altitude RMSE for the TOC-matching trajectory predictions are substantially less for every one of the ten most common aircraft types in Fort Worth Center compared to the baseline.



**Fig. 13 Altitude RMSE by aircraft type for trajectory predictions generated at 18,000 ft (5-minute prediction time).**

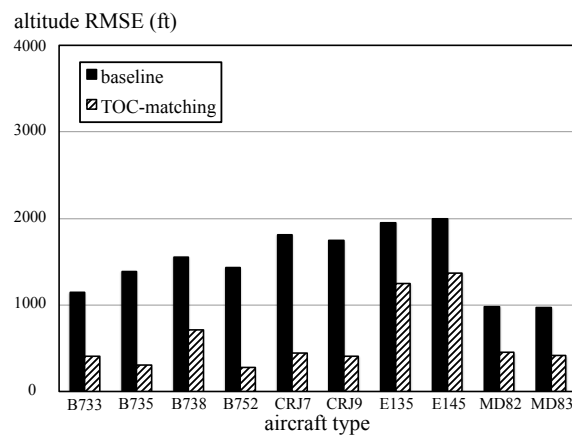
These ten aircraft types comprise 88% of the analysis data set (see Table 2). The TOC-matching method reduced the altitude RMSE of Embraer E135 and E145 flights the least (about 30%). Their TOC-matching altitude RMSE of 2206 ft and 2410 ft, respectively, are more than double the vertical separation standard of 1000 ft. A more in-depth investigation of Embraer E145 flights is presented in the next section to identify the underlying causes.

**Table 2 Altitude RMSE for trajectory predictions at 18,000 ft by aircraft type (5-minute prediction time)**

Aircraft Type	Altitude RMSE (baseline, ft)	Altitude RMSE (TOC-matching, ft)	Altitude RMSE Reduction (%)	% of Data Set (~1350 Flights)
B733	1218	656	46.1	5.5
B735	1605	510	68.2	1.9
B737	2664	845	68.3	2.0
B738	2025	951	53.0	4.5
B752	1815	1004	44.7	2.2
CRJ7	2387	834	65.1	4.6
E135	3247	2206	32.1	7.6
E145	3501	2410	31.2	16.3
MD82	1166	600	48.5	31.4
MD83	1134	577	49.1	12.2
Top 10	2157	1350	37.4	88.2
Top 10 excluding, E135, E145	1465	673	54.0	63.9
All Flights	2113	1310	38.0	100.0

The altitude RMSE of the TOC-matching trajectories were at or less than the vertical separation standard of 1000 ft for each of the eight other most common aircraft types in Fort Worth Center. The aggregate altitude RMSE of the TOC-matching trajectory predictions for these eight aircraft types was 673 ft, which was 54% less than the baseline of 1465 ft. In particular, the altitude RMSE for Boeing 737-500 (B735), Boeing 737-700 (B737), and Bombardier CRJ7 were all reduced by at least 65%. This analysis demonstrates that the TOC-matching method can substantially decrease altitude trajectory prediction errors for climbing flights in Fort Worth Center towards what is required for higher levels of automation in separation assurance.

A similar analysis of altitude RMSE by aircraft type for trajectory predictions made at 24,000 ft on a 5-minute look-ahead time (see Figure 14) found that the TOC-matching method became more effective at higher altitudes as flights approached TOC. E135 and E145 flights still had the smallest improvement at 36% and 31%, respectively, while the other eight aircraft types had an overall improvement of about 60%, which is six percentage points higher than at 18,000 ft.



**Fig. 14 Altitude RMSE by aircraft type for trajectory predictions generated at 24,000 ft (5-minute prediction time).**

## 2. *Percentage of Flights with Absolute Altitude Error Greater than 1000 Feet*

The TOC-matching method also substantially decreased the percentage of flights whose absolute altitude errors exceeded the vertical separation standard of 1000 ft. For trajectory predictions generated at 18,000 ft, this metric for the aggregate data set at a 5-minute prediction look-ahead time decreased by about 25 percentage points from 54.7% for the baseline trajectories to 29.6% for the TOC-matching method trajectories (see Table 2). The most substantial

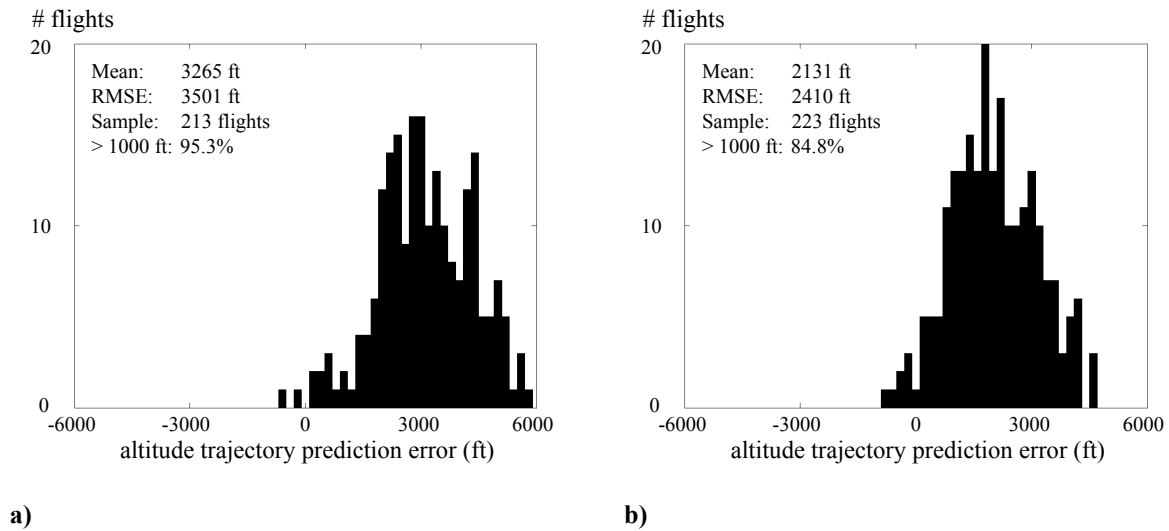
reductions among the 10 most common Fort Worth Center aircraft types were observed for the B737 and CRJ7 at 68% and 57%, respectively, which are consistent with the results in the previous section. Likewise, the smallest decreases were observed for the E135 and E145 at 15.0% and 10.6%, respectively. These results reinforce the need to further analyze the E145 flights to identify the underlying causes.

**Table 3 Percentage of flights with absolute altitude error greater than 1000 ft for trajectory predictions at 18,000 ft by aircraft type (5-minute prediction time)**

Aircraft Type	% with Absolute Error > 1000 ft (baseline)	% with Absolute Error > 1000 ft (TOC-matching)	Percentage Point Difference	% of Data Set (~1350 Flights)
B733	47.4	14.5	32.9	5.5
B735	50.0	3.8	46.2	1.9
B737	89.3	21.4	67.9	2.0
B738	70.5	32.8	37.7	4.5
B752	43.3	13.3	30.0	2.2
CRJ7	76.2	19.0	57.1	4.6
E135	93.9	78.8	15.0	7.6
E145	95.3	84.8	10.6	16.3
MD82	30.5	8.4	22.1	31.4
MD83	31.5	7.8	23.7	12.2
Top 10	44.2	31.0	13.3	88.2
Top 10 excluding, E135, E145	41.1	11.7	29.5	63.9
All Flights	54.8	29.6	25.3	100.0

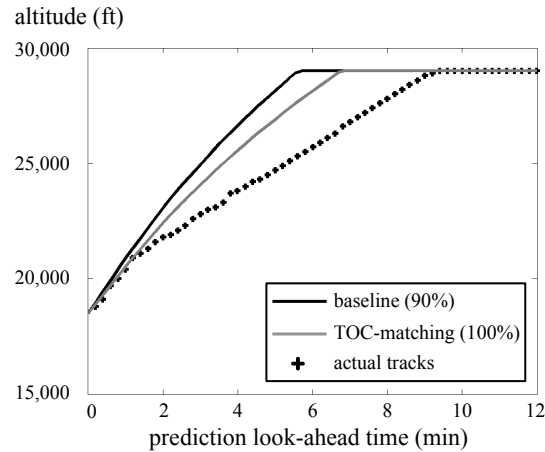
### C. Closer Investigation of E145 Flights

E145 flights were analyzed further because the TOC-matching method improved the climb trajectory prediction accuracy of these flights the least out of the ten most common aircraft types in Fort Worth Center. Figures 15a and 15b are histograms of the altitude errors of the baseline and TOC-matching trajectory predictions, respectively, at 18,000 ft for E145 flights. One interesting observation is that these error distributions have similar shapes. The main difference is that the distribution of the TOC-matching errors is closer to zero than the baseline. By comparison, their aggregate-level counterparts (see Figures 7a and 7b) are both centered substantially closer to zero and have different shapes, with the baseline errors spread out and the TOC-matching errors tightly clustered around zero with a tail on the positive end. This tail is largely comprised of E135 and E145 flights, which further reinforces the fact that these two aircraft types are atypical in terms of the effectiveness of the TOC-matching method on Fort Worth Center climbing departures.



**Fig. 15 Altitude errors for a) baseline, and b) TOC-matching trajectory predictions generated at 18,000 ft (E145 flights, 5-minute prediction time).**

One particular E145 flight is illustrated in Figure 16 because its behavior appeared to be representative of E145 flights in general. Its baseline and TOC-matching trajectory predictions had altitude errors that were within 100 ft of the mean errors in Figures 15a and 15b, respectively. The baseline and TOC-matching trajectory predictions for this flight both have faster climb rates than the actual track after about 1 minute. In fact, the latter prediction's TOC is substantially different than the actual TOC even though the TOC-matching method correctly selected the candidate trajectory prediction generated using the largest weight percentage (i.e., 100% of the maximum gross takeoff weight in the CTAS E145 aircraft performance model), which had the slowest climb rate.



**Fig. 16 Example E145 flight track data and trajectory predictions generated at 18,000 ft.**

Note how different the climb profile of the actual flight is compared to the baseline and TOC-matching trajectory predictions starting at around the 1-minute mark. This implies that simply extending the upper limit of the weight range of the TOC-matching method for E145 flights cannot improve its effectiveness to the same extent as the other common aircraft types in Fort Worth Center. More specifically, it indicates that it is necessary to augment the TOC-matching method to use additional flight parameters besides weight for generating candidate trajectory predictions to improve its effectiveness for E145 aircraft in particular and climbing flights in general. These results suggest that significant improvements in trajectory prediction accuracy for climbing E145 flights can be achieved if climb profile data (e.g., climb speed schedule) were made available in addition to TOC time.

With regard to the trajectory predictor itself, these results also indicate that it is worth exploring adjustments to the CTAS E145 aircraft performance model and/or utilizing the E145 model in other databases, such as the Base of Aircraft Data (BADA) [27], to improve the effectiveness of the TOC-matching method for E145 flights. In fact, CTAS was recently enhanced with the capability to use BADA models instead of CTAS models for aircraft types specified by the user [28]. Evaluation using actual Fort Worth Center traffic showed that trajectory predictions for E145 flights had smaller errors when the BADA model was used. This could improve the effectiveness of the TOC-matching method for E145 flights by increasing the likelihood that the TOC of one of the candidate climb trajectory predictions will more closely correspond to the TOC data provided. Follow-up work is needed to identify all of the aircraft types that should use BADA models instead of CTAS models for the TOC-matching method.

## V. Discussion

The substantial improvement in climb trajectory prediction accuracy that was achieved by the TOC-matching method for the CTAS trajectory predictor is promising. This section begins with a discussion on why it can also be used in any airspace and with any trajectory predictor including the FAA's ERAM TP [22] whose aggregate errors are comparable to the baseline CTAS errors [5]. Following that are two distinct complementary analyses of the TOC-matching method. The first is a sensitivity analysis of algorithm effectiveness to TOC data accuracy because the FMS-predicted TOC times that flights can broadcast are unlikely to be their actual TOC times. The second is an evaluation of the algorithm's effectiveness if substantially smaller subsets of 6 and 11 of the standard 51 candidate trajectory predictions can be generated (due to computational constraints, for example). Of course, the use of the TOC-matching method is predicated upon TOC data being available. The last part of this section will describe how ADS-B Out could potentially enable sharing of flight data such as TOC.

### A. General Applicability

The TOC-matching method presented and analyzed in this study was evaluated using actual Fort Worth Center traffic data and the CTAS trajectory predictor, but it can be utilized more generally in any airspace for all aircraft types (even those not present in current operations) and with any trajectory predictor. Given how the TOC-matching method was able to improve climb trajectory prediction accuracy for the wide range of commercial and regional aircraft types in Fort Worth Center (see the analysis by aircraft type in Section IV.B), it is expected to be effective in other Centers and for other aircraft types as well. Based on the analysis of E145 flights in Section IV.C, the extent to which this is true depends on numerous factors, including the departure procedures of flights in those Centers and how closely the baseline CTAS TS can model the climb profile actually flown.

The TOC-matching method can also be used with any trajectory predictor. Unlike the adaptive weight [9, 11] and adaptive thrust algorithms [16-17] that also improved trajectory prediction accuracy for climbing flights, the TOC-matching method does not require that the trajectory predictor use the standard point-mass equations of motion in a kinetic approach. Rather, it can also be applied to kinematic trajectory predictors like the FAA's ERAM TP [22] by using parameters besides aircraft weight, such as vertical rate and/or climb speed schedule, to generate candidate trajectory predictions. No fundamental changes to the ERAM TP would be required. Thus, compared to the adaptive weight algorithm, the TOC-matching method appears to be a more viable approach for improving climb trajectory

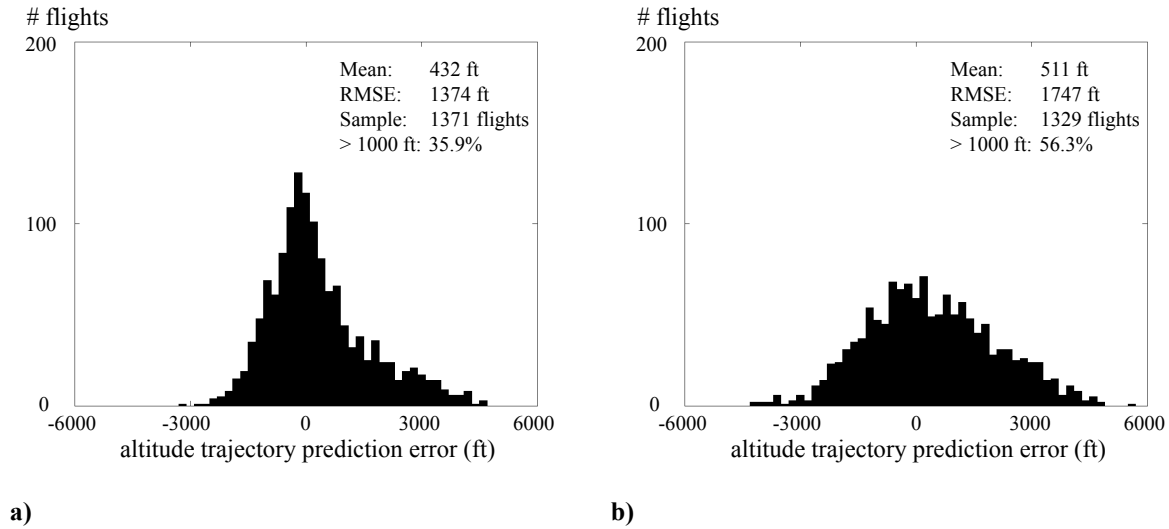


prediction accuracy in ERAM from its current performance level [5] toward what is required for higher levels of automation in separation assurance.

## **B. Sensitivity to TOC Data Accuracy**

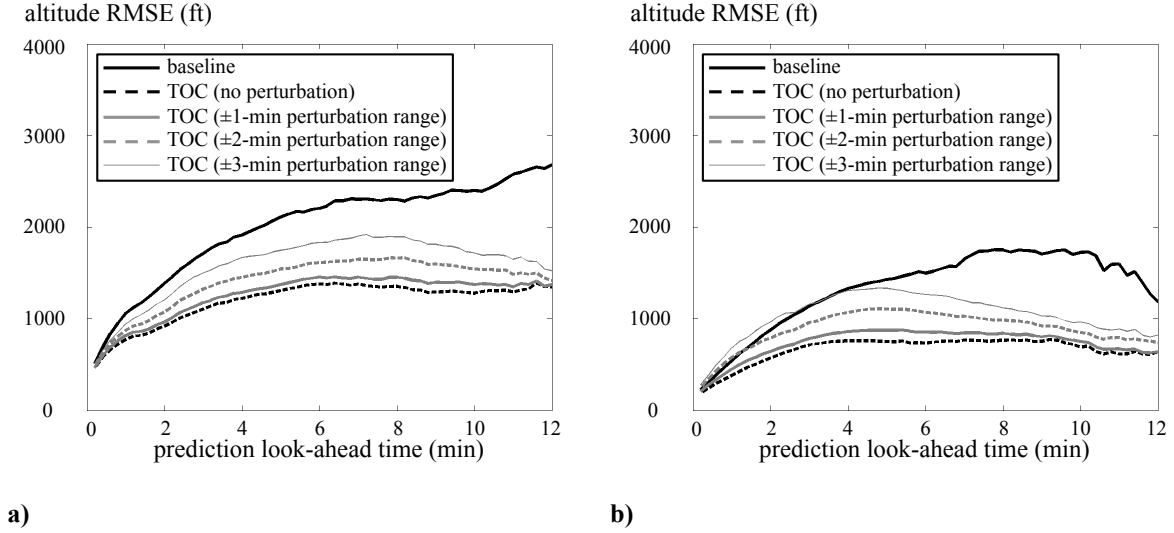
The improvement in climb trajectory prediction accuracy that was achieved using error-free (actual) TOC data is promising and yet an overestimation of what could be expected in actual operations because there will be error in the FMS-predicted TOC times that flights broadcast. As such, it is important to conduct a sensitivity analysis where the TOC-matching method uses perturbed TOC data to determine the extent to which it can improve climb trajectory prediction accuracy under less-than-ideal conditions. Uniform distributions for the perturbations of  $\pm 1$  minute,  $\pm 2$  minutes, and  $\pm 3$  minutes were used based on engineering judgment due to a lack of published data.

The sensitivity analysis was performed on the same data set used in previous sections. As expected, the level of improvement in climb trajectory prediction accuracy achieved by the TOC-matching method decreases as the range of perturbations applied to the TOC data increases. For instance, when the TOC-matching method was used with no perturbations, the altitude RMSE for trajectory predictions made at 18,000 ft was 1309 ft on a 5-minute prediction look-ahead time (see Figure 7b). By comparison, the altitude RMSE was 1374 ft in the  $\pm 1$ -minute perturbation range case and 1747 ft in the  $\pm 3$ -minute perturbation range case (see Figures 17a and 17b). Note that both are still less than the baseline altitude RMSE of 2111 ft (see Figure 7a).



**Fig. 17 Altitude errors for TOC-matching trajectories with a)  $\pm 1$ -minute perturbation range, and b)  $\pm 3$ -minute perturbation range generated at 18,000 ft (aggregate, 5-minute prediction time).**

Similar results were observed across all prediction look-ahead times for the trajectory predictions generated at 18,000 ft. The reduction in altitude RMSE ranged from 4% at 12 seconds in the  $\pm 3$ -minute perturbation range case to 50% at 12 minutes for the no-perturbation case (see Figure 18a). For trajectory predictions made at 24,000 ft (see Figure 18b), the altitude RMSE were also less at all prediction look-ahead times when the range of perturbations was  $\pm 1$  minute (thick gray line) or zero (dashed black line). However, the altitude RMSE were marginally greater than the baseline at look-ahead times up to about 1 minute in the  $\pm 2$ -minute perturbation range case (dashed gray line) and up to around 3 minutes in the  $\pm 3$ -minute perturbation range case (thin gray line). This is because TOC data errors have greater effect at 24,000 ft when flights are closer to TOC than at 18,000 ft. However, compared to the baseline, the altitude RMSE of the TOC-matching trajectory predictions in the  $\pm 3$ -minute perturbation range case was marginally greater by 142 ft at most. On the other hand, it was lower than the baseline at all look-ahead times beyond 3 minutes by as much as 783 ft.



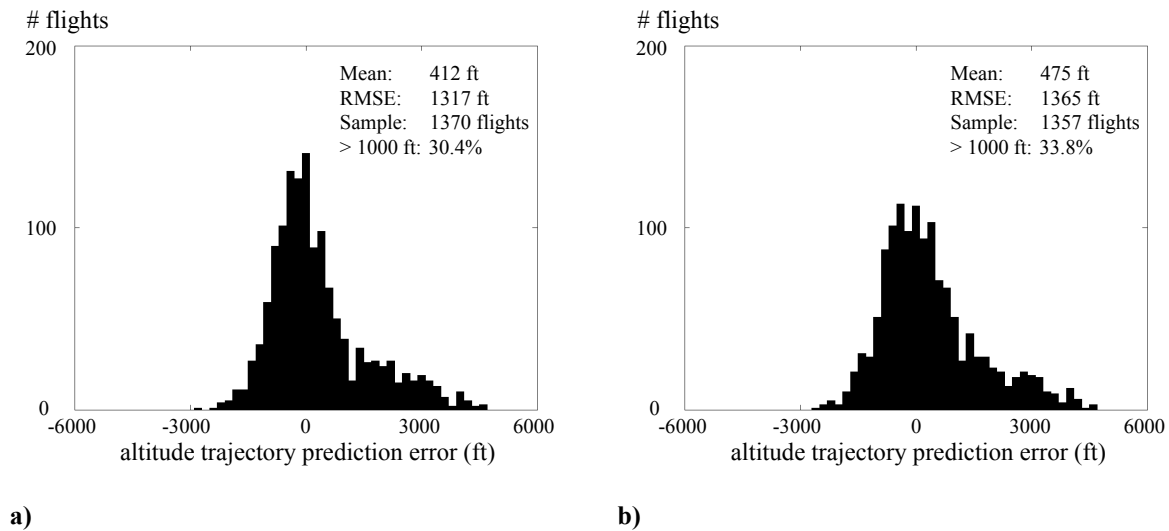
**Fig. 18 Sensitivity analysis of TOC-matching method effectiveness to TOC data accuracy for trajectory predictions generated at a) 18,000 ft, and b) 24,000 ft.**

### C. Sensitivity to Fewer Candidate Trajectory Predictions

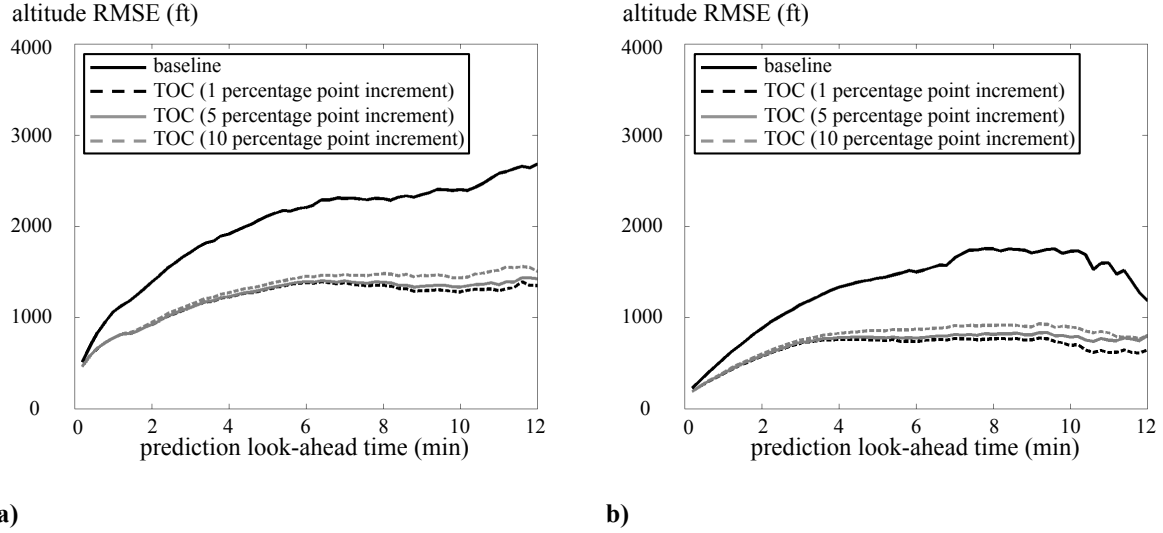
The TOC-matching method may also be less effective if the complete set of 51 candidate trajectory predictions for each climbing flight cannot be generated in real time. In this study using CTAS and its TS trajectory predictor, there were no difficulties computing trajectory predictions for a full Fort Worth Center traffic load. However, if the TOC-matching method causes runtime performance issues for any reason, the number of candidate trajectory predictions that are computed and analyzed may need to be reduced. Utilizing a smaller, moving range (e.g.,  $\pm 10$  percentage points of the most recent weight percentage) because the TOC-matching weight is stable during most of the climb phase (see Figure 4) and/or utilizing larger increments between the modeled weights used to generate the candidate trajectory predictions are two possible mitigations.

The latter was analyzed using the same data as in previous sections for increments of 5 and 10 percentage points from 50% to 100% of the maximum modeled gross takeoff weight. The result is that 11 and 6 candidate trajectory predictions, respectively, were computed for each flight instead of the standard 51. As expected, the TOC-matching method improved climb trajectory prediction accuracy less when larger increments were used, but the change was relatively small. For instance, with the complete set of 51 candidate trajectory predictions, the altitude RMSE of the TOC-matching method trajectory predictions at 18,000 ft was 1309 ft for a 5-minute prediction look-ahead time (see

Figure 7b). For weight increments of 5 and 10 percentage points (see Figures 19a and 19b), the altitude RMSE only increased marginally to 1317 ft and 1365 ft, respectively. By comparison, the baseline altitude RMSE was 2111 ft (see Figure 7a). The TOC-matching method was also robust to smaller sets of candidate trajectory prediction across all prediction look-head times and at different altitudes throughout the climb phase as seen in Figures 20a and 20b.



**Fig. 19** Altitude errors for TOC-matching trajectories using modeled weight parameter increments of a) 5 percentage points, and b) 10 percentage points generated at 18,000 ft (aggregate, 5-minute prediction time).



**Fig. 20** Sensitivity analysis of TOC-matching method effectiveness to modeled weight increment size for trajectory predictions generated at a) 18,000 ft, and b) 24,000 ft.

#### D. Automatic Dependent Surveillance-Broadcast (ADS-B) Out

The TOC-matching method is promising, but it can only be utilized to improve trajectory prediction accuracy for climbing flights if TOC data is made available. One potential method for flights to share this information is through ADS-B Out. The FAA has mandated that all aircraft operating in transponder airspace be equipped with ADS-B Out by January 1, 2020 [20]. It is expected to improve flight track data quality and trajectory prediction accuracy since it enables one-second updates of horizontal position and velocity, barometric and geometric altitude, and vertical rate.

ADS-B Out can also facilitate the sharing of an aircraft's short-term flight intent data via Target State Reports of the horizontal and vertical targets for its active flight segment and/or its long-term flight intent data via Trajectory Change Reports [21]. Extending the ADS-B Out mandate to require the broadcast of at least one of these reports would enable the use of the TOC-matching method (or a similar algorithm). Policymakers and stakeholders will need to weigh the benefits of improved trajectory prediction accuracy on the level of safety and efficiency that can be provided during separation assurance against the cost of enhancing and certifying the FMS and corresponding databus to export this information.

#### E. Communication Frequency Congestion Concerns

The trajectory prediction error analysis conducted in this study indicate that the TOC-matching method must be extended to utilize additional flight parameters besides weight for generating candidate trajectory predictions in order to reduce trajectory prediction errors for climbing flights towards what is required for higher levels of robust automation in separation assurance. With regard to trajectory prediction, it is of course desirable to have as much up-to-date information about flights and their current operating conditions as possible. However, communication frequency congestion constraints may limit the amount and type of data that can be broadcast by aircraft to be used by the trajectory predictor.

The investigation into the large trajectory prediction errors for E145 flights (relative to the other common aircraft types in Fort Worth Center) suggested that significant reductions could be achieved if climb profile data (e.g., climb speed schedule) were made available in addition to TOC time. By comparison, the results of existing studies indicate that having additional information on environmental conditions, such as wind, would not have as much of an effect.

A 2006 study by Mondoloni investigated the effect of wind uncertainty on trajectory prediction accuracy [29]. He developed and verified a statistical model for wind uncertainty that included wind-shear error, which especially affects aircraft during climb and descent. When the statistical model of wind uncertainty was applied to a model of a large jet with various combinations of 1) fast/slow target speed, and 2) heavy/light load, the effect on altitude and along-track error during climb was small on average (less than 200 ft and 1 nmi for all prediction look-ahead times up to the 12-minute maximum look-ahead time analyzed for the TOC-matching method). In addition, the TOC data used in the TOC-matching method already includes the effect of wind (and the various other sources of uncertainty that affect aircraft climb trajectories). This means that the TOC-matching method already implicitly compensates for wind uncertainty to some extent.

In summary, prior research demonstrated the effect of wind uncertainty on trajectory prediction errors in both the vertical and horizontal dimensions is small on average. As such, if data communication frequency congestion is a constraint that limits the amount and type of flight and environmental data that can be shared with the trajectory predictor, the TOC-matching method would be improved to a greater extent by having flight intent information, such as climb speed schedule, than by having wind.

## **VI. Conclusions**

Substantial reductions in trajectory prediction errors for climbing flights are readily achievable if just one more flight parameter is made available: TOC time. This information could potentially be shared in NextGen via ADS-B Out. TOC time is more likely to be shared by airlines compared to proprietary flight-specific data that could also be used to improve climb trajectory prediction accuracy, such as aircraft weight, thrust, and climb profile.

This study develops a straightforward algorithm that utilizes 51 different modeled aircraft weights to compute a set of candidate trajectory predictions for each climbing flight and selects the trajectory prediction whose TOC is closest to the TOC provided in terms of time. It can be used with any trajectory predictor, including the one in the FAA's En Route Automation Modernization system for the Next Generation Air Transportation System.

The TOC-matching method was evaluated on a data set comprised of over 1000 actual departures in Fort Worth Center. It decreased the altitude root mean square error for climb trajectory predictions made at 18,000 ft by 38% on a 5-minute prediction look-ahead time. This is nearly double the 20% decrease in altitude RMSE that was achieved by the adaptive weight algorithm [11] for the same trajectory predictor in an analysis of departures in Fort Worth Center as in this study. Furthermore, the percentage of flights with absolute altitude error greater than the vertical separation standard of 1000 ft was reduced from 55% to 30%.

Out of the ten most common aircraft types in Fort Worth Center, the Embraer E135 and E145 had the smallest improvement at 30% because the climb profiles of these flights were typically outside of the range of climb profiles spanned by the candidate trajectory predictions. By comparison, the 5-minute altitude root mean square error for each of the other eight most common aircraft types in Fort Worth Center dropped to or below the vertical separation standard of 1000 feet with an overall decrease of 54%. A closer investigation of the E145 flights indicated that: 1) simply extending the upper limit of the weight range of the TOC-matching method for E145 flights cannot improve its effectiveness to the same extent as the other common aircraft types in Fort Worth Center, 2) the algorithm may need to be extended to utilize additional flight parameters besides weight for generating candidate trajectory predictions, and 3) significant improvements in trajectory prediction accuracy for climbing E145 flights can be achieved if climb profile data (e.g., climb speed schedule) were made available in addition to TOC time.

Sensitivity analyses of the TOC-matching method determined that it is robust to both TOC time data errors and fewer candidate trajectory predictions (the latter may be necessary due to computational constraints, for example). Although more elegant and/or efficient methods are possible, this study demonstrates proof of concept that having

TOC time data can substantially improve trajectory prediction accuracy for climbing flights toward what is required for higher levels of automation in separation assurance.

### **Acknowledgments**

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